Short Answer Question

Ques① Explain synthetic division of an algebraic equation by linear factors.

Ques② State the four general theorems concerned with real roots of an algebraic equation.

Ques③ Explain order of convergence for iterative methods and bisection method.

Ques④ Find a root of the equation $f(x) = x^3 - 4x - 9 = 0$, using the bisection method upto four iterations.

Ques⑤ Explain the Method of iteration and, Solve $x = 0.21 \sin(0.5 + x)$ by Iteration method upto four decimal places. Starting with $x_0 = 0.12$.

Ques⑥ Find the root of the equation $2x = \cos x + 3$ correct upto three decimal places by Iteration method.

Ques⑦ Describe the method of False Proportion.

Ques⑧ Find a root of the equation $x - e^x = 0$ correct upto three decimal places by the secant method.

Ques⑨ Find the real root of the equation $x^3 - 9x + 1 = 0$ by Regula-Falsi Method upto four decimal places.

Ques⑩ Describe and Obtain Newtons formulae for determining special type of Roots.

Ques 11 Find a real root of the equation $3x = \cos x + 1$ by Newton - Raphson's method.

Ques 12 Describe solutions of Simultaneous Algebraic equations and their three criterion for consistency.

Ques 13 Solve by Gauss - Elimination Method the following system of equations:-

$$6x + 3y + 2z = 6$$
$$6x + 4y + 3z = 0$$
$$20x + 15y + 12z = 0$$

Ques 14 Solve the following system of equations by LU - Decomposition Method.

$$2x_1 + x_2 + x_3 = 2$$
$$x_1 + 3x_2 + 2x_3 = 2$$
$$3x_1 + x_2 + 2x_3 = 2$$

Ques 16 Describe Doolittle Method.

Ques 17 Using the triangularisation Method, find the inverse of the matrix $A = \begin{bmatrix} 50 & 107 & 36 \\ 25 & 54 & 20 \\ 31 & 66 & 21 \end{bmatrix}$

Ques 18 State and Prove the difference theorem of factorial notation.

Ques 19 Evaluate (i) $\Delta \tan^{-1} x$ (ii) $\Delta^2 \cos 2x$.

Ques 20 Describe Newton's forward interpolation.

Long Answer Questions

Ques 1 a) Describe Bisection Method.

b) Using Bisection Method, find a real root of the equation upto two decimal places :- $f(x) = 3x - \sqrt{1+\sin x} = 0$

Ques 2 (a) Prove that Bisection Method is always convergent

(b) Find a real root of the equation $x \log_{10} x = 1.2$ by bisection method upto two decimal places.

Ques 3 (a) If α, β are the roots of $x^2 + ax + b = 0$, show that the iteration $x_{n+1} = -\left[\frac{ax_n + b}{x_n}\right]$ will converge near $x = \alpha$ if $|\alpha| \geq |\beta|$ and the iteration $x_{n+1} = \frac{-b}{x_n + \alpha}$ will converge near $x = \alpha$ if $|\alpha| < |\beta|$.

(b) Find the root of the equation $2x = \cos x + 3$. correct to three decimal places, by Iteration method.

Ques 4

(a) Find a real root of the equation $x^3 - x^2 - 2 = 0$ by Regula-Falsi method upto three decimal places.

(b) Determine the root of the equation $f(x) = \cos x - xe^x = 0$ using the secant method upto four decimal places.

Ques 5 (a) Using Newton-Raphson's formula find the real root (b) of the following equations upto four decimal places

(a) $x \log_{10} x = 1.2$ (b) $x^2 - 5x + 2 = 0$

Ques 6 (b) Describe Gauss-Elimination Method

(a) Using Gauss-Elimination method solve

$$2x_1 + 4x_2 + x_3 = 3$$
$$3x_1 + 2x_2 - 2x_3 = 2$$
$$x_1 - x_2 + x_3 = 6$$

Ques 7 (b) Describe LU Decomposition Method.

(a) Solve the following by LU Decomposition Method

$$2x - 3y + 10z = 3$$
$$-x + 4y + 2z = 20$$
$$5x + 2y + z = -12.$$

Ques 8 (a) If the interval of differencing is 'h', the prove that $\Delta^n x^{(n)} = n!\, h^n$ and $\Delta^{n+1} x^{(n)} = 0$

(b) Prove that $x^{(-n)} = \frac{1}{(x+n)^{(n)}}$, if $h = 1$

Ques 9 (a) Let $f(x) = a_0 x^n + a_1 x^{n-1} + \ldots + a_n, a_0 \neq 0$ be a polynomial degree n, then prove that $\Delta^n f(x) = n!\, a_0 h^n$, where 'h' being an interval of differencing. Also $\Delta^{n+1} f(x) = 0$.

(b) If $\Delta^3 u_x = 0$, Prove that

$$u_{x+\frac{1}{2}} = \frac{1}{2}(u_x + u_{x+1}) - \frac{1}{16}(\Delta^2 u_{x+1} + \Delta^2 u_x)$$

Ques 10 The following table gives the marks secured by 100 students in the Numerical Analysis subject:

Range of marks	30-40	40-50	50-60	60-70	70-80
No. of students	25	35	22	11	7

Use Newton's forward interpolation formula to find:

(a) The number of students who got more than 55 marks.

(b) The number of students who secured marks in the range from 36 to 45.

Multiple Choice Questions

1) If $f(x)$ is _____ in the closed interval $[a, b]$, then the bisection method is not applicable.
(a) Continuous (b) Not Continuous
(c) ~~divisible~~ convergent (d) None of these.

2) $f(x)$ does not have a real root, if $f(x)$ is _____
~~(a) Continuous~~ in a closed interval $[a, b]$ & does not cut the x-axis
(a) Continuous (b) Not continuous
(c) ~~Divisible~~ convergent (d) None of these

3) The number of _____ roots cannot exceed the number of variations in $f_n(-x)$.
(a) real (b) imaginary (c) positive (d) Negative.

4) The iterative method is applicable if _____ for $x \in (a, b)$
(a) $|\phi'(x)| < 1$ (b) $|\phi(x)| < 1$ (c) $|\phi'(x)| > 1$ (d) $|\phi(x)| > 1$

5) In _____ method two points x_0 and x_1 are taken such that $f(x_0)$ and $f(x_1)$ are of opposite signs, i.e., $f(x_1)f(x_0) < 0$.
(a) Iteration Method (b) Bisection Method
(c) Regula-Falsi Method (d) Secant Method

6) Newton's Method converges if
(a) $|f'(x)f''(x)| > |f(x)|^2$
(b) $|f'(x)f''(x)| < |f(x)|^2$
(c) $|f(x)f'(x)| < |f''(x)|^2$
(d) $|f(x)f''(x)| < |f'(x)|^2$

7) Newton's formula to determine inverse of 'a' is:

(a) $x_{n+1} = x_n(2 - ax_n)$ (b) $x_{n+1} = \frac{1}{2}(x_n + \frac{a}{x_n})$

(c) $x_{n+1} = \frac{1}{2}x_n(3 - ax_n^2)$ (c) $x_{n+1} = x_n\left(\frac{p+1 - ax_n^p}{p}\right)$

8) Newton's formula to determine square root of 'a' is:

(a) $x_{n+1} = x_n(2 - ax_n)$ (b) $x_{n+1} = \frac{1}{2}(x_n + a/x_n)$

(c) $x_{n+1} = \frac{1}{2}x_n(3 - ax_n^2)$ (d) $x_{n+1} = x_n\left(\frac{p+1 - ax_n^p}{p}\right)$

9) Newton's formula to determine Inverse square root of 'a' is:

(a) $x_{n+1} = x_n(2 - ax_n)$ (b) $x_{n+1} = \frac{1}{2}(x_n + a/x_n)$

(c) $x_{n+1} = \frac{1}{2}x_n(3 - ax_n^2)$ (d) $x_{n+1} = x_n\left(\frac{p+1 - ax_n^p}{p}\right)$

10) Newton's formula to determine general formula for p^{th} root of 'a' is:

(a) $x_{n+1} = x_n(2 - ax_n)$ (b) $x_{n+1} = \frac{1}{2}(x_n + a/x_n)$

(c) $x_{n+1} = \frac{1}{2}x_n(3 - ax_n^2)$ (d) $x_{n+1} = x_n\left(\frac{p+1 - ax_n^p}{p}\right)$

11) A system of simultaneous Algebraic Equation are consistent and have a unique solution if

(a) rank (A) ≠ rank [A/B] = n (the order of A)

(b) rank (A) ≠ rank [A/B] > n (the order of A)

(c) rank (A) = rank [A/B] < n (the order of A)

(d) None of these

12) A system of simultaneous Algebraic Equation are inconsistent and have no solution if

(a) rank (A) ≠ rank [A/B] = n (the order of A)

(b) rank (A) ≠ rank [A/B] > n (the order of A)

(c) rank (A) = rank [A/B] < n (the order of A)

(d) None of these.

13) A system of simultaneous Algebraic Equation are consistent and has an infinite number of solutions if
(a) rank $(A) \neq$ rank $(A/B) = n$ (the order of A)
(b) rank $(A) \neq$ rank $(A/B) > n$ (the order of A)
(c) rank (A) = rank $(A/B) < n$ (the order of A)
(d) None of these.

14) _____ method can also be called factorization method
(a) LU Decomposition Method
(b) Doolittle Method
(c) Triangularization Method
(d) All of these.

15) For the operator Δ which of the following is not true?
(a) $\Delta[f(x) \pm g(x)] = \Delta f(x) \pm \Delta g(x)$
(b) $\Delta[c f(x)] = c\, \Delta f(x)$, c being constant
(c) $\Delta(c) \neq 0$, c being constant.
(d) None of these

16) If $y_1 - y_0 = \underset{\text{operator}}{\delta y_{1/2}}$, $y_2 - y_1 = \delta y_{3/2}, \ldots, y_n - y_{n-1} = \delta y_{n-1/2}$ then the difference is called
(a) Forward difference operator
(b) Backward difference operator
(c) Central difference operator
(d) Shift operator.

17) If $\Delta y_k = y_{k+1} - y_k$ then the difference operator is called
(a) Forward difference operator
(b) Backward difference operator
(c) Central difference operator
(d) Shift operator

18) If $\nabla y_k = y_k - y_{k-1}$ then the difference operator is called
(a) Shift operator
(b) Average operator
(c) Central difference operator
(d) Backward difference operator

19) If $E^n y_x = y_{x+nh}$ then the difference operator is called
(a) Shift operator
(b) Average operator
(c) Central difference Operator
(d) Backward difference operator

20) Which of the following relations is false?
(a) $\Delta = E - 1$ (b) $\delta = E^{1/2} + E^{-1/2}$
(c) $\nabla = 1 - E^{-1}$ (d) $\mu = 1/2\,(E^{1/2} + E^{-1/2})$